

HOSTOS COMMUNITY COLLEGE
DEPARTMENT OF MATHEMATICS

Math 150 SI **College Algebra with Trigonometric Functions**

CREDIT HOURS: 4.0

EQUATED HOURS: 4.0

CLASS HOURS: 7.5

PREREQUISITE: Placement via the CUNY's Proficiency Index for Elementary Algebra

RECOMMENDED TEXT:

The College Algebra Book, Custom Edition, Terence Brenner, Daniel Maysonet, Nieves Angulo, Gisselle Guzman, Pearson Custom Mathematics, Pearson Learning Solutions

COURSE DESCRIPTION:

This course introduces the concept of mathematical functions in preparation for further studies in pre-calculus and calculus. The course content includes an in-depth treatment of the following topics: operations with real numbers, polynomial functions and factoring techniques, rational functions and equations, radical functions and equations, complex numbers, quadratic equations, graphs of linear equations, system of equations, inequalities and quadratic functions, applications to geometry, conic sections and an introduction to the study of trigonometric functions. This course is appropriate for liberal arts students as well as STEM majors.

STUDENT LEARNING OUTCOMES:

1. Interpret and draw appropriate inferences about functions, conic sections and their properties from quantitative representations such as graphs of basic algebraic functions and graphical representation of conic sections.
2. Use algebraic, numerical and graphical methods to solve mathematical problems including representing functions as graphs and graphical representations of complex numbers and conic sections.
3. Represent quantitative problems expressed in natural language in suitable algebraic, functional and graphical form.
4. Effectively communicate solutions to mathematical problems in written, graphical or equation form.
5. Evaluate solutions to problems and verify the validity of graphs of functions and conics properties for reasonableness by inspection.

6. Apply mathematical methods to problems in other fields of study such as Basic Engineering, Physics and Geometry

EXAMINATIONS: A minimum of five partial tests (suggested 15% each) and a comprehensive departmental final examination (suggested 30%)

GRADES: A, A⁻, B⁺, B, B⁻, C⁺, C, D, I, F.

LEARNING OUTCOMES ASSESSMENT TOOLS:

SLO#1:

- Unit Test #1: Differentiate between functions and relations, determine symmetry, locate asymptotes and perform reflections and translations of graphs of functions.
- Unit Test #4 Inferences relating the graphs of conic sections with properties such as foci, vertices, symmetry and center.
- Departmental Final Exam: Cumulative

SLO#2:

- Unit Test #1: Use algebraic and numerical methods to determine the inverse of a function. Find the composition of functions. Verify inverses by composition and by reflection. Determine the graph of a function through use of translations and reflections. Utilize a table of values for polynomial, rational, and radical functions.
- Unit Test #2: Represent complex numbers graphically
- Unit Test #3: Determine a table of values for a trigonometric function.
- Unit Test #4: Construct graphs of conic sections.
- Departmental Final Exam: Cumulative

SLO#3:

- Unit Test #1: Translate word problems involving modeling with functions
- Unit Test #3: Convert word problems into appropriate trigonometric functions.
- Departmental Final Exam: Cumulative
- Unit Test #4: Translate conic section properties expressed in natural language into correct graphical representations

SLO#4:

- Unit Test #1: Express solutions to problems using appropriate written, graphical or functional methods.
- Unit Test #2: Communicate solutions to rational, radical and complex number problems in accurate and appropriate form.
- Unit Test #3: State solutions to trigonometric problems using appropriate properties or table of values.
- Unit Test #4: Solve application problems involving Free Falling Objects, Projectiles, Pythagorean Theorem and Area and Volume Problems.
- Departmental Final Exam: Cumulative

SLO#5:

- Unit Test #1: Use the vertical and horizontal line tests to examine graphs to determine if they represent functions or relations.
- Unit Test #3: Express trigonometric values in appropriate units. Inspect trigonometric functions for appropriate domains and ranges.
- Unit Test #4: Use graphical representation of conic sections to verify their defining properties such as foci, radii, vertices, asymptotes, etc.
- Departmental Final Exam: Cumulative

SLO#6:

- Unit Test #1: Use functions to model business applications.
- Unit Test #3: Apply trigonometry to problems in Physics
- Unit Test #4: Apply quadratic equations to solve applications in Physics and Right Triangle Geometry.
- Departmental Final Exam: Cumulative

COURSE OUTLINE

CLASS

TOPICS

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R1. THE REAL NUMBER SYSTEM AND INTRODUCTION TO ALGEBRA (3 sessions)

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| 1 | Introduction: sets of integers, rational, irrational and real numbers, complex numbers: The real number line, absolute value, comparing, ordering ($<$, $=$, $>$), and trichotomy (<i>e.g.</i> , $-1 < 0 < 5$), addition of real numbers and properties of addition, subtraction of real numbers |
| 2 | Multiplication of real numbers and properties of multiplication: division of real numbers, powers of real numbers with natural number exponents, order of operations, including multiple grouping symbols and exponents |
| 3 | The concept of variable and constant; defining like and unlike terms. Definition of algebraic expressions, numerical coefficient (including implied 1 and -1), literal part, monomial, binomial, and polynomial. The substitution principle for evaluating formulas and algebraic expressions. |

R2. LINEAR EQUATIONS AND INEQUALITIES IN ONE VARIABLE, TOGETHER WITH APPLICATIONS (7 sessions)

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| 4 | Simplifying expressions by combining like terms. Using the distributive property to remove parentheses (expansion). Using the distributive property to |
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combine like terms (contraction). Definition and solution of a linear equation in one variable. Solving linear equations using: Addition/Subtraction and Multiplication/Division. Principles of Equality. Language translation problems (e.g., “three less than twice a number is what?”)

- 5-6 Solving a linear equation involving parentheses, fractions, and decimals; identities and contradictions
- 7 Solving and graphing linear inequalities
- 8 Solving word problems (application problems) using linear equations: Solving literal equations for a given variable, including perimeter and area formulas. Translating from English to algebra, (e.g., “15 is 12 less than 2 times a number”, “30 subtracted from 7 times a number is 4”)
- 9 More algebra word problems, averages, perimeter and area problems, and linear inequality problems
- 10 **General review of R1 & R2:** which should include at least the following: order of operations, substitution, signed numbers, translation problems, solving linear equations, and solving and graphing linear inequalities

R3. EXPONENTS AND OPERATIONS WITH POLYNOMIALS AND SPECIAL PRODUCTS (6 sessions)

- 11 The 7 exponent rules, including negative exponents (no rational exponents)
- 12 Addition and subtraction of polynomials
- 13 Multiplication of polynomials: A monomial times a monomial, including exponents and variables. A monomial times a polynomial (the distributive property). A binomial times a binomial: FOILing
- 14 Multiplication of polynomials: A binomial times a binomial: Special Products. Perfect square trinomials. Difference of two squares. A binomial times a trinomial (e.g., $(3x - 5)(x^2 - 6x + 4)$)
- 15 Division of Polynomials: A monomial by a monomial. A polynomial by a monomial, where quotient has no remainder (*i.e.*, no division by more than a monomial)
- 16 Review for Test 1 (R1, R2, R3)
- 17 **Exam 1**

Chapter 1. FACTORING (5 sessions):

- 18 Factor, prime factor, and greatest common factor; Factoring a polynomial that has a common factor in each of its terms; Factoring by grouping, with up to 4 terms, & terms with multiple variables (e.g., $45cw + 63cz - 20dw - 28dz$)
- 19 Factoring trinomials of the form $x^2 + bx + c$.
- 20 Factoring trinomials of the form $ax^2 + bx + c$.
- 21 Factoring perfect square trinomials; Factoring the difference of two squares; Factoring difference and sum of cubes. Factoring completely, including multi-step problems such as $36x^2y - 100y^3$ (GCF + difference of two squares)
- 22 Solving quadratic equations by factoring, including multi-step factoring (e.g., $x^2 + 2x = 15$).

Chapter 2. RATIONAL EXPRESSIONS & EQUATIONS
(7 sessions)

- 23 Rational Expressions: Definition, Domains, Simplifying
- 24 Multiplication and Division of Rational Expressions
- 25 Addition and Subtraction of Rational Expressions: Least Common Denominator
- 26 Simplifying Complex Rational Expressions
- 27 Solving Rational Equations That Reduce to Either Linear Equations or Quadratic Equations That Can Be Solved by Factoring
- 28 Word Problems Involving Rational Expressions
- 26 Review for Exam 2 (Chap 1, Chap 2)
- 27 **EXAM 2**

Chapter 3. RATIONAL EXPONENTS AND RADICALS
(9 sessions)

- 28 Radical Expressions: Definition of a Root, Principal Roots, Finding Real Roots of Perfect Powers
- 29 Simplifying Radical Expressions: Monomial Radicands and the Product Rule, Rational Radicands and the Quotient Rule
- 30 Operations on Radicals with the Same Index: Addition, Subtraction, Multiplication and Division

- 31 Multiplication and Division of Radical Expressions, Rationalizing Denominators
- 32 Solving Radical Equations That Reduce to Either Linear Equations or Quadratic Equations That Can Be Solved by Factoring, Applications of the Pythagorean Theorem
- 33 Rational Exponents, Multiplication and Division of Radicals with Different Indices
- 34 Complex Numbers: the Imaginary Unit i , Pure Imaginary Numbers, Powers of i , Square Roots of Monomials Revisited
- 35 Operations on Pure Imaginary Numbers, Definition of Complex Numbers, Equivalent Complex Numbers, Solving Basic Complex Equations
- 36 Operations on Complex Numbers, Graphical Representation of Complex Numbers
- 37 **Chapter 4. QUADRATIC EQUATIONS** (6 Sessions)
- 38 Quadratic Equations: Solving by Factoring, Solving Using the Square Root Property
- 39 Solving Quadratic Equations by Completing the Square
- 40 The Quadratic Formula, The Discriminant and the Nature of the Roots of a Quadratic Equation
- 41 Properties of the Roots of a Quadratic Equation, Determining a Quadratic Equation Given Its Roots
- 42 Applications: Number Relations, Free Falling Objects, Projectiles, Pythagorean Theorem, Area Problems
- 43 Review for Test 3 (Chapter 3, Chapter 4)
- 44 **Test 3**
- Chapter 5. EQUATIONS IN TWO VARIABLES** (7 sessions):
- 45 The Cartesian coordinate system; Ordered pairs of real numbers and finding points in a plane, given a table
- 46 Definition and solution of a linear equation in two variables ($ax+by=c$). Graphing a linear equation: By finding two points, By the x - and y -intercepts method, Equations of horizontal and vertical lines (e.g., find the equation of the

horizontal line passing through the point $(-5,3)$; find the equation of the vertical line passing through the point $(-5,-2)$

- 47-48** Concept of the slope of a straight line: Slope formula. Finding the slope of a line on a graph given its equation. Finding the slope of a line using $y=mx+b$. Given possible graphs of a line, use slope and y-intercept to select correct graph. Slopes of horizontal and vertical lines (slopes of lines parallel to the x- and y-axes).
- 49** Finding equations of lines: Using the slope-intercept formula ($y=mx+b$). Using the point-slope formula. Given two points on the line
- 50** Systems of two linear equations in two variables, including rational coefficients: Three possibilities: intersecting lines, parallel lines, coincident lines.
- 51** Solving System of Linear Equations by Substitution method and Addition method

Chapter 6. FUNCTIONS (5 Sessions)

- 52** Concept of a Function: Domain, Range, Vertical Line Test, Using Function Notation
- 53** Graphing Functions from Equations and Tables of Data
- 54** Algebraic Operations on Functions and Composition of Functions
- 55** Inverses of Functions
- 56** Review for Test 4 (chapter 5, chapter 6)
- 57** **Test 4**

Chapter 7. INTRODUCTION TO TRIGONOMETRY (5 Sessions)

- 58** Introduction to Trigonometry: Defining Angles, Degree Measure, Types of Angles, Relationships Between Angles
- 59** Radian Measure, Arc Length, Area of a Circular Sector, Conversion Between Degrees and Radians
- 60** Similar Triangles and Applications
- 61** Right Triangle Trigonometry, Trigonometric Functions of 30° , 45° and 60°
- 62** Solving Right Triangles and Applications;

Chapter 8. CONIC SECTIONS (5 Sessions)

- 63** Conic Sections: The Distance Formula and the Circle, Center and Radius, Standard Form, Sketching the Graph
- 64** The Parabola: Definition, Concavity, Intercepts, the Vertex, Axis of Symmetry, Standard Form, Sketching the Graph
- 65** The Ellipse: Definition, Standard Form, Sketching the Graph
- 66** The Hyperbola: Definition, Asymptotes, Standard Form, Sketching the Graph
- 67** Solving Systems of Equations in Two Variables Consisting of a Linear Equation and a Quadratic Equation;
- 68** Review for Exam 4 (chapter 7, chapter 8)
- 69** **Test 5**
- 70** **Review for Final Exam**