

**HOSTOS COMMUNITY COLLEGE  
DEPARTMENT OF MATHEMATICS AND COMPUTER SCIENCE**

MAT 217                                      Linear Algebra

CREDIT HOURS:                              4.0

EQUATED HOURS:                              4.0

CLASS HOURS:                                  4.0

PREREQUISITE:                                MAT 210 Calculus I

PRE/COREQUISITE:                            MAT 220 Calculus II

**RECOMMENDED TEXTS:**

- Anton, H., Rorres, C., Elementary Linear Algebra. Applications Version. 9<sup>th</sup> Ed., Wiley, 2005.
- Lay, D. Linear Algebra and its Applications. 4<sup>th</sup> Ed., Pearson, 2012.
- Leon, S. Linear Algebra with Applications. 8<sup>th</sup> Ed., Pearson, 2010.
- Hefferon, J. LINEAR ALGEBRA. Available for free download at <http://joshua.smcvt.edu/linearalgebra>

EXAMINATIONS:                              A minimum of three partial tests and a comprehensive final examination

GRADES:                                        A, A-, B+, B, B-, C+, C, D, INC, F

**COURSE DESCRIPTION:**

This course introduces the concepts and methods of solution of systems of linear equations with an arbitrary number of equations and variables by using both the elimination and matrix methods; algebra of matrices; determinants; vector spaces and subspaces, norm of a vector and distance between vectors; linear dependence and independence; basis and dimension of vector spaces, orthogonal and orthonormal bases, change of basis; linear transformations and their matrices, kernel and image; real inner products, eigenvalues and eigenvectors; diagonalization of symmetric matrices and its application to quadratic forms. During the course, students will be trained to use technology to solve linear algebraic problems. The technological means include Mathematica, Maple, Matlab, Sage, or graphing calculator TI-89 or equivalent. Not Available for Students who have taken MAT 320 Linear Algebra with Vector Analysis

## Math 217 (Linear Algebra) Student Learning Outcomes

After completion of the course, the students will:

1. Be able to solve linear systems with arbitrary numbers of variables and equations by the elimination method. Be able to find a general solution and to analyze its structure by dividing variables into free and basic. Understand the relationship between linear systems and matrices.
2. Be able to perform algebraic operations over matrices to find the inverse matrix. Know how to solve a linear system by performing operations with matrices.
3. Know the definition and the properties of determinants. Be able to calculate determinants by reducing the order and by the elimination method. Be able to solve systems of linear equations by Cramer's Rule. Know how to find the determinant of the product of two matrices, of the inverse matrix, and of the transpose matrix.
4. Know the definition and the main examples of vector spaces. Be able to determine if a set of given vectors are linearly dependent. Know the definition of the dimension of a vector space, and be able to determine it. Understand what the subspace is and be able to construct the spanning subspace of a given set of vectors. Be able to determine if a vector belongs to a subspace by solving a system of linear equations. Know how to find a basis of a vector space, to change the basis and to find the vector coordinates in a new basis.
5. Know the definition and main examples of vector spaces with an inner product. Understand the relationship between an inner product and the norm of a vector, and angle and distance between two vectors. Know how to calculate norms, angles and distances between two vectors given an inner-product. Know the definition and understand the property of orthogonality in vector spaces with inner-product; know the generalized Pythagorean Theorem.
6. Know the definition and main examples of linear transformations of vector spaces. Be able to determine the kernel and the image of a linear transformation. Know the definition of a matrix of a linear transformation. Know the definition of eigenvalues and eigenvectors, and be able to find them by solving a characteristic equation or a system of corresponding linear equations. Understand the meaning and the process of matrix diagonalization. Be able to diagonalize a symmetric matrix. Understand the equivalence of two matrices and interpretation of the Jordan normal form.
7. Be familiar with applications of linear - algebraic methods to the problems in different fields of study, such as geometry, biology, chemistry, physics, business, and economics.

## LEARNING OUTCOMES ASSESSMENT TOOLS:

### SLO#1:

- Unit Test #1: Identify if the linear system is consistent. Find the unique or general solution, by using the elimination method. Identify the matrix of a linear system by matrix methods.
- Unit Test #2: Apply linear systems to the investigation of vector spaces.
- Unit Test #3: Devise a linear system and find its general and partial solutions to determine eigenvalues and eigenvectors.

### SLO#2:

- Unit Test #1: Find the inverse matrix by the elimination method. Solve a system of linear equations by the matrix method and by the Cramer's Rule.
- Unit Test #2: Use matrix operations to investigate vector spaces.
- Unit Test #3: Find the matrix of a linear transformation. Devise a matrix of the linear system needed to find eigenvalues and eigenvectors.

### SLO#3:

- Unit Test #1: Determine whether the system is consistent by finding the determinant of its matrix or in the process of using the Cramer's Rule.
- Unit Test #2: Use the determinant to find an area of a parallelogram or a volume of a parallelepiped spanning over vectors in a plane or in space.
- Unit Test #3: Find the eigenvalues that provide the zero value for the determinant of the linear system.

### SLO#4:

- Unit Test #1: Use the properties of matrices for the investigation of the vector spaces.
- Unit Test #2: Determine if a set of vectors is linearly independent. Determine the dimension and find a basis of the vector space.
- Unit Test #3: Identify the vector space, and the subspaces corresponding to the eigenvectors.

### SLO#5:

- Unit Test #1: Not applicable.
- Unit Test #2: Not applicable.
- Unit Test #3: Identify the inner product. Use it to normalize the eigenvectors.

### SLO#6:

- Unit Test #1: Not applicable.
- Unit Test #2: Not applicable.
- Unit Test #3: Identify the transformation of the vector space and its linearity. Find the kernel and the image, eigenvalues and eigenvectors. Determine, whether the eigenvectors are orthogonal. Form the eigenvectors' basis\*.

SLO#7:

- Unit Test #1: Solve word problems using a linear system. Use a matrix method for solving word problems.
- Unit Test #2: Use the notion of the vector space when formalizing word problems.
- Unit Test #3: Use eigenvectors as basis vectors to diagonalize a symmetric matrix of the quadratic form. Show the role of eigenvalues in the transformed quadratic form.

## SUGGESTED COURSE OUTLINE

CLASS	TOPIC
<b>SYSTEMS OF LINEAR EQUATIONS</b>	
1	Systems of linear equations with arbitrary number of variables.
2-4	Linear systems in matrix form. Elementary row operations. Row echelon and reduced row echelon forms. Solution by the Gauss-Jordan elimination. General solution, basic and free variables.
5-6	Matrix operations. Null, identity, and diagonal matrices and their roles in matrix algebra. Matrices of the elementary row operations.
7-8	Invertible matrices. Finding the inverse matrix by row operations. Matrix methods of solution of linear systems. LU factorization.*
<b>DETERMINANTS</b>	
9-11	Determinants. Definition of the determinant as a function of a square matrix*. Properties of determinants. Evaluating determinants by row reduction. Cramer's rule.
12-13	Determinants of inverse, product and transpose* of matrices. Computation of the inverse of a matrix by using determinants.
14	REVIEW for Exam I
15	EXAM I
<b>VECTOR SPACES</b>	
16-17	Definition and examples of vector spaces
18	Linear dependence and independence of vectors. Dimension of a vector space.
19-20	Bases of a vector space. Subspaces and spanning sets. Change of the basis. Finding if a given vector belongs to a subspace.
21-22	Spaces spanned by the columns and the rows of a matrix. Equality of their dimensions; equality of the row and column ranks.
<b>REAL INNER-PRODUCT SPACES</b>	
23-24	Definitions and examples of spaces with inner product. Distances and angles in Euclidean and general inner-product spaces. Gram-Schmidt orthogonalization process*
25	REVIEW for Exam II
26	Exam II

LINEAR TRANSFORMATIONS	
27	Definition and examples of linear transformations.
28-29	Kernel and image of linear transformations.
30-32	Matrix representation of linear transformations. Images of basis vectors. Matrices of rotations, dilations, and reflections.* Matrices of the transformations preserving lengths, areas, or angles.*
EIGENVALUES AND EIGENVECTORS	
33-35	The characteristic equation. Eigenvalues and eigenvectors of linear transformations.
36-37	Diagonal matrices. Block diagonal matrices.* Diagonalization of a matrix.* Jordan normal form.* Equivalent matrices.* Quadratic forms and their matrices. Orthogonal diagonalization*.
38	REVIEW for Exam III
39	EXAM III
40-42	REVIEW for Final

\*Suggested topics